The Early Days of Interactive Proofs

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The Death of Proof

Computers are transforming the way mathematicians discover, prove and communicate ideas, but is there a place for absolute certainty in this brave new world?

By John Horgan


Berkeley, California, USA
1985

A Model-Theoretic Analysis of Knowledge: Preliminary Report

Ronald Fagin
Joseph Y. Halpern
Moshe Y. Vardi[1]

IBM Research Laboratory
San Jose, CA 95193

SIAM J. COMPUT.
Vol. 18, No. 1, pp. 186-208, February 1989

012

THE KNOWLEDGE COMPLEXITY OF INTERACTIVE PROOF SYSTEMS*

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Abstract. Usually, a proof of a theorem contains more knowledge than the mere fact that the theorem is true. For instance, to prove that a graph is Hamiltonian it suffices to exhibit a Hamiltonian tour in it; however, this seems to contain more knowledge than the single bit Hamiltonian/non-Hamiltonian.

In this paper a computational complexity theory of the "knowledge" contained in a proof is developed. Zero-knowledge proofs are defined as those proofs that convey no additional knowledge other than the correctness of the proposition in question. Examples of zero-knowledge proof systems are given for the languages of quadratic residuosity and quadratic nonresiduosity. These are the first examples of zero-knowledge proofs for languages not known to be efficiently recognizable.

Key words. cryptography, zero knowledge, interactive proofs, quadratic residues

AMS(MOS) subject classifications. 68Q15, 94A60

Interactive Proofs

All Powerful

Computationally
Limited

JOURNAL OF COMPUTER AND SYSTEM SCIENCES **36**, 254–276 (1988)

Arthur–Merlin Games: A Randomized Proof System, and a Hierarchy of Complexity Classes

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Eötvös University, Budapest, Hungary and University of Chicago, Chicago Illinois

AND

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Technion, Haifa, Israel

Received June 24, 1986; revised August 3, 1987

1986

Interactive Proofs

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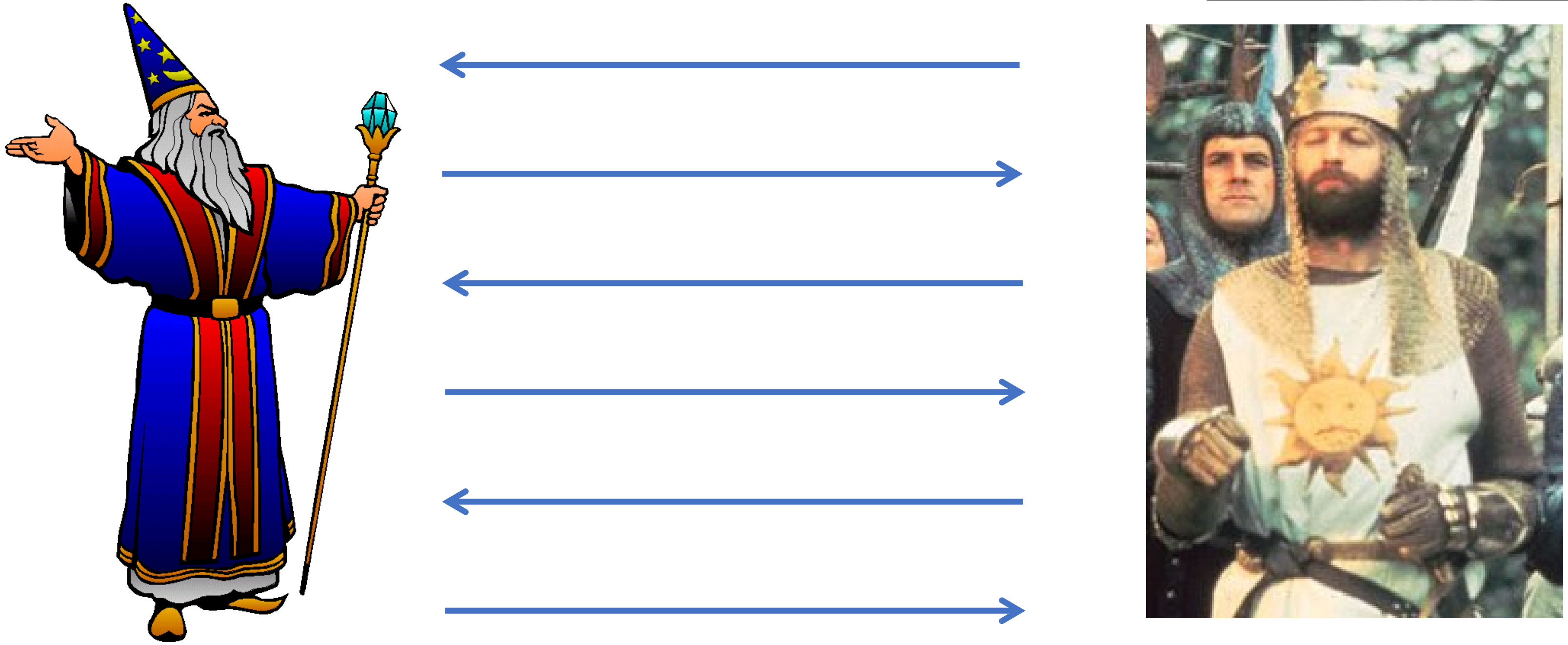
Goldwasser-Sipser
Public vs Private Coins



All Powerful

Computationally Limited

How to Prove All NP Statements in Zero-Knowledge
and
a Methodology of Cryptographic Protocol Design

(Extended Abstract)

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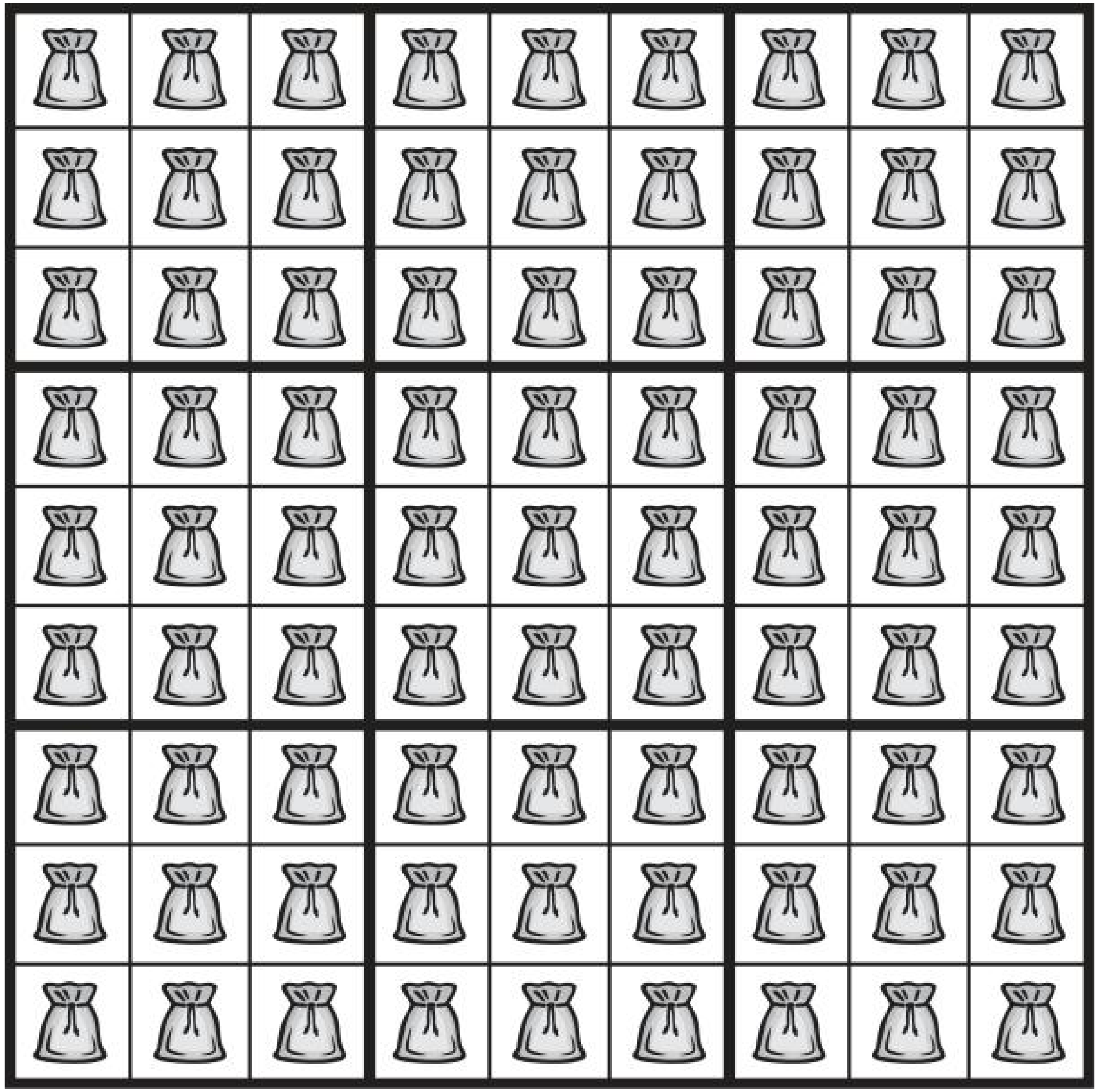

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Old	New
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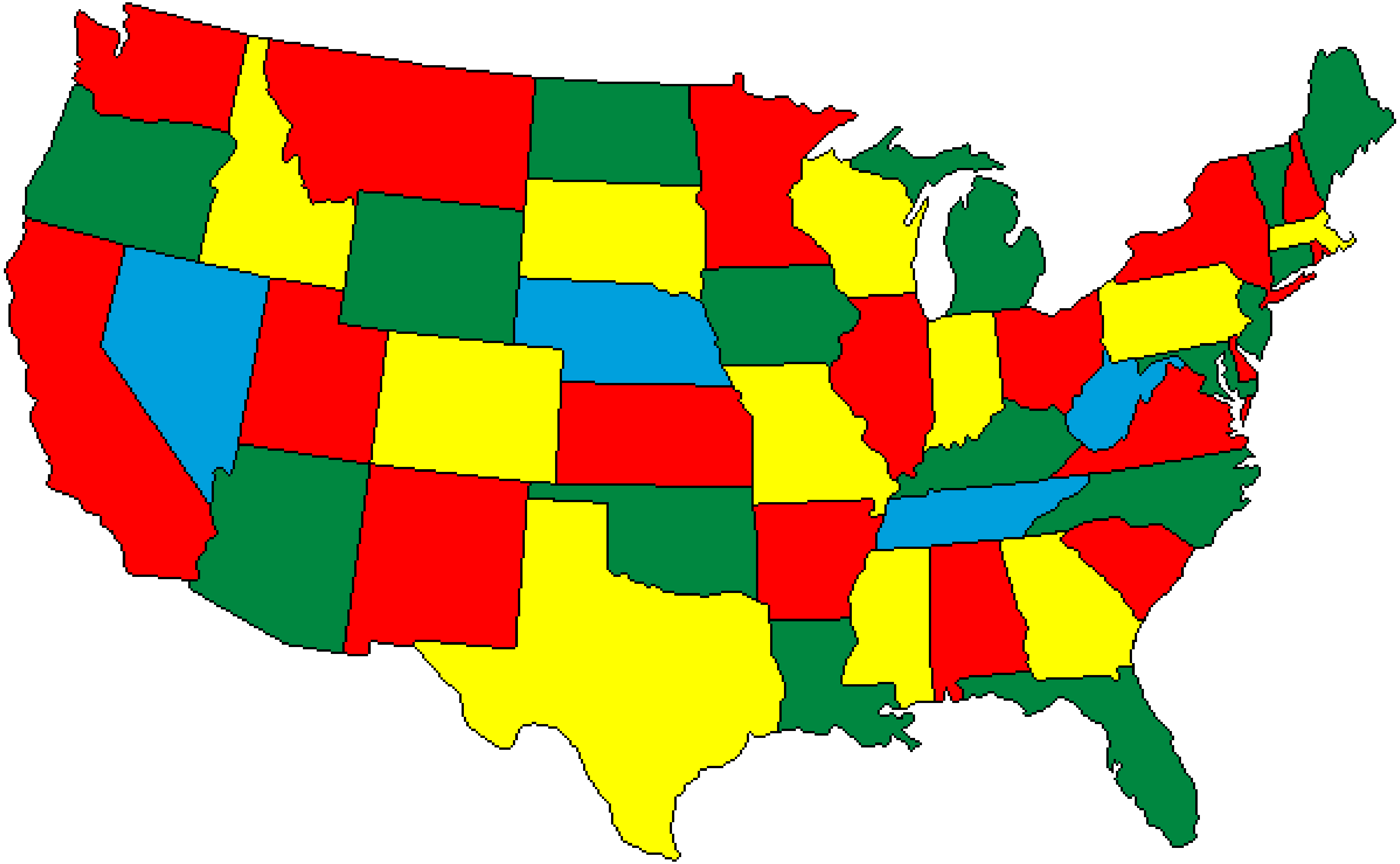

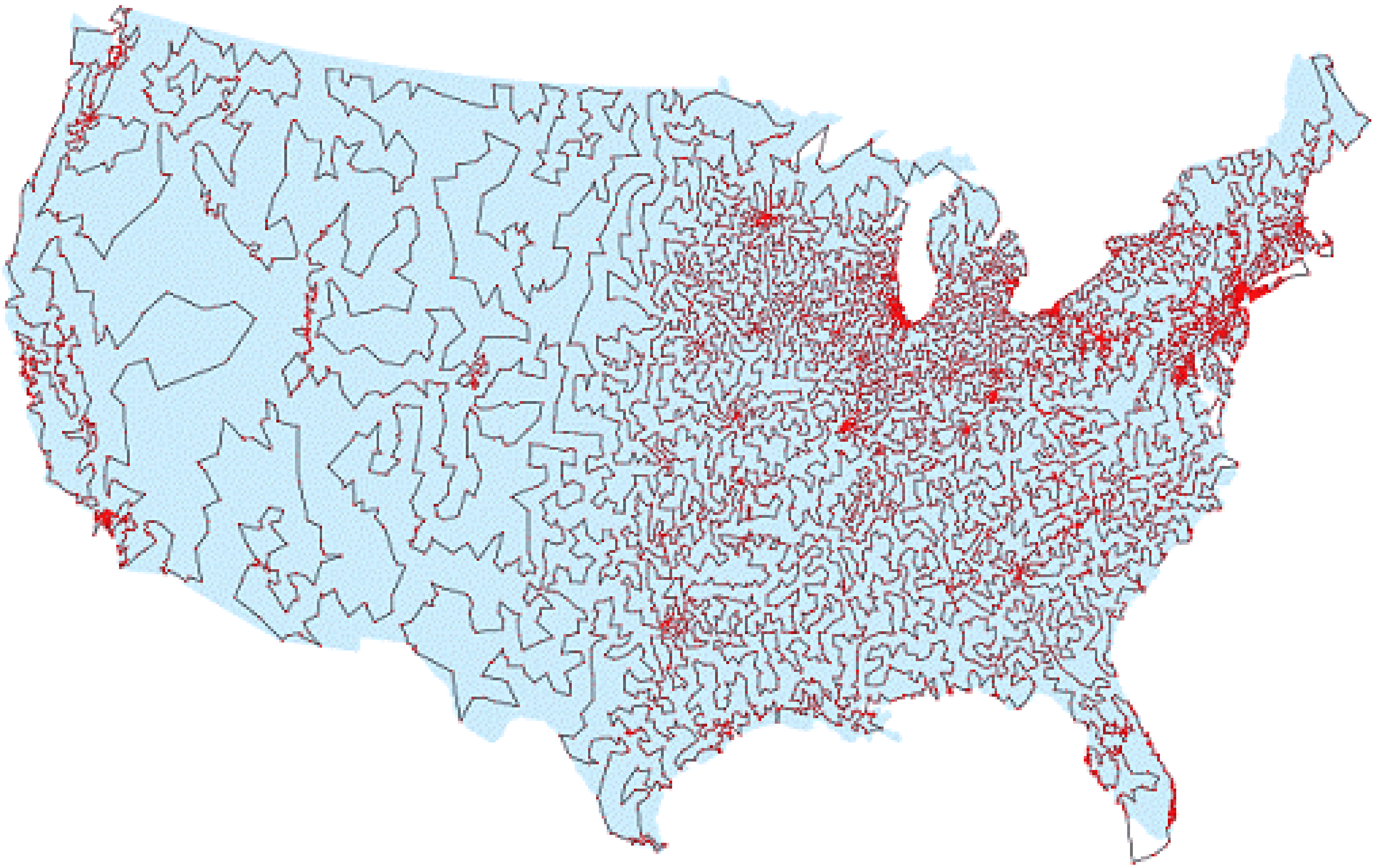

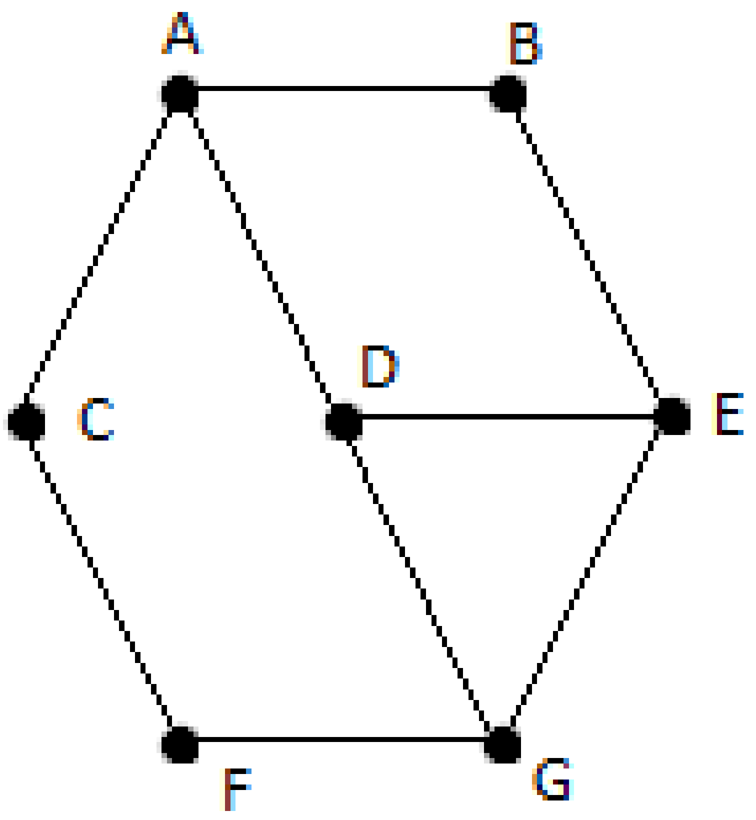
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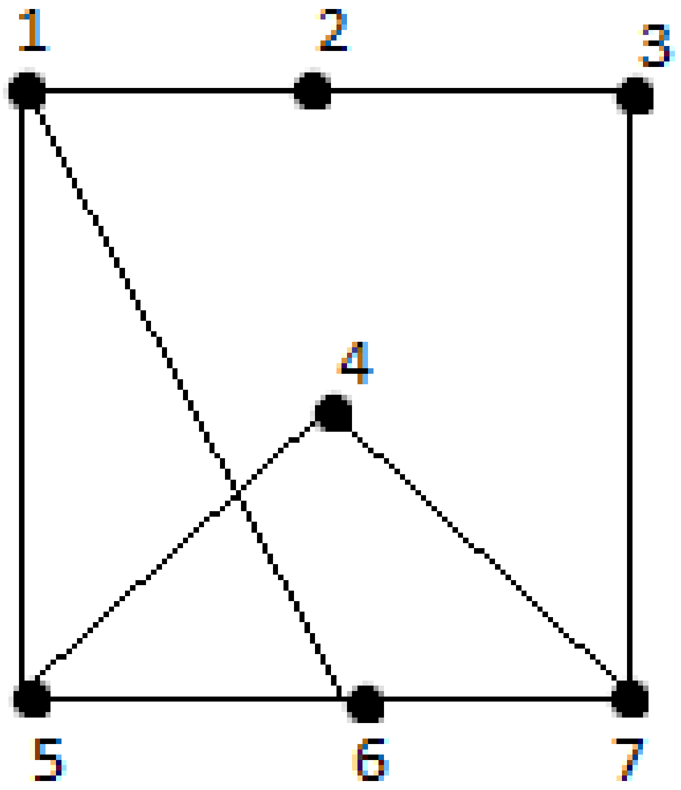
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1987

The Complexity of Perfect Zero-Knowledge

(extended abstract)

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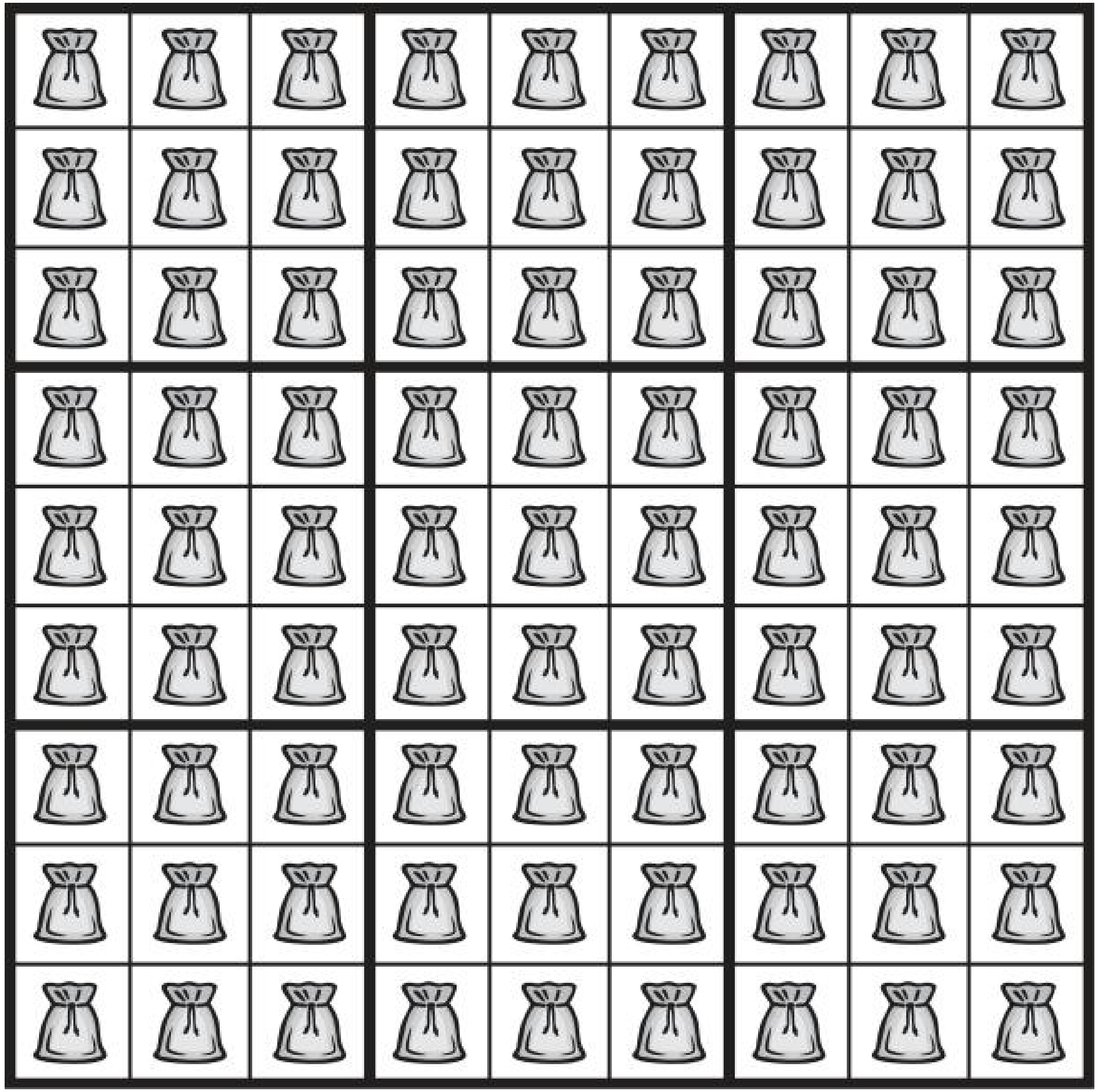

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7	6	5	2	4	9	3	8	1

1988

Multi-Prover Interactive Proofs:
How to Remove Intractability Assumptions

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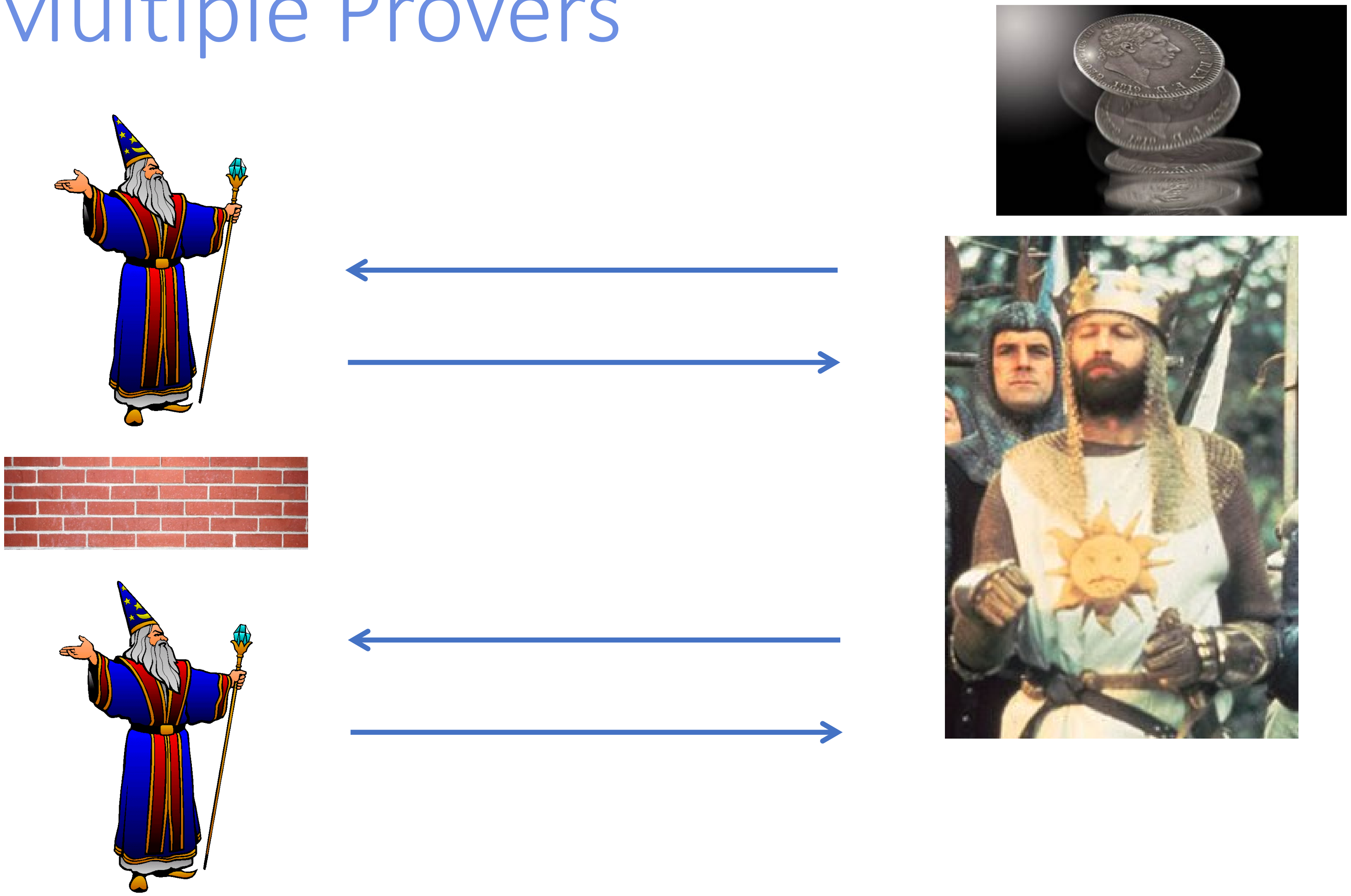
Interactive Proofs

All Powerful

Computationally Limited

Multiple Provers



On the Power of Multi-Prover Interactive Protocols

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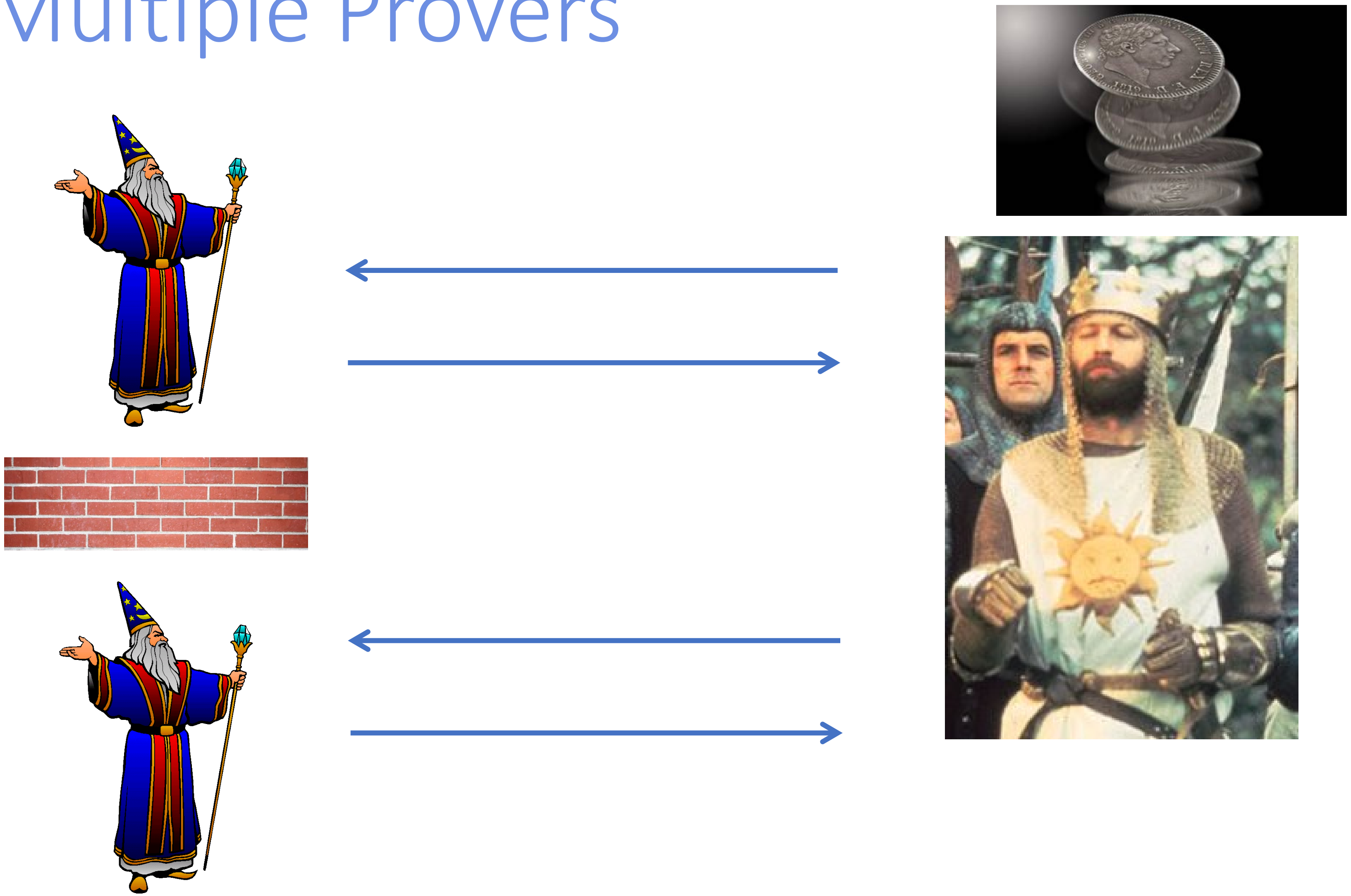
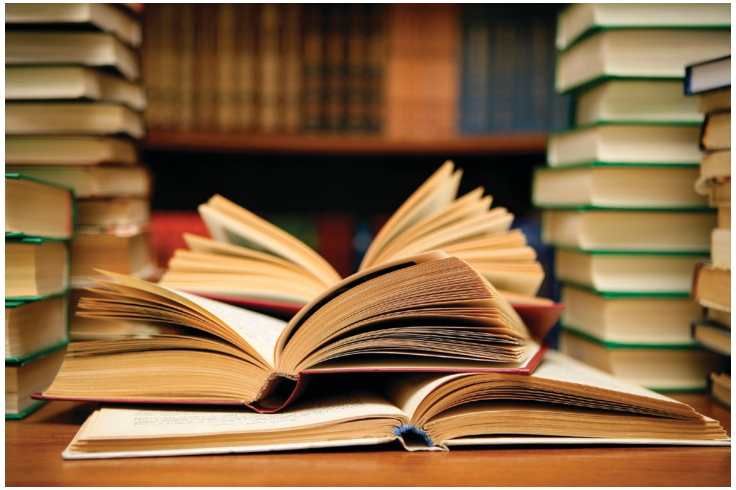
Multiple Provers



Probabilistically Checkable Proof



A PARALLEL REPETITION THEOREM*

RAN RAZ†

Abstract. We show that a parallel repetition of any two-prover one-round proof system (MIP(2, 1)) decreases the probability of error at an exponential rate. No constructive bound was previously known. The constant in the exponent (in our analysis) depends only on the original probability of error and on the total number of possible answers of the two provers. The dependency on the total number of possible answers is logarithmic, which was recently proved to be almost the best possible [U. Feige and O. Verbitsky, *Proc. 11th Annual IEEE Conference on Computational Complexity*, IEEE Computer Society Press, Los Alamitos, CA, 1996, pp. 70–76].

1989

Annual Symposium on Theoretical Aspects of Computer Science

STACS 1990: STACS 90 pp 37-48 | Cite as

Hiding instances in multioracle queries

Authors | Authors and affiliations

Donald Beaver, Joan Feigenbaum

R. Lipton. New directions in testing. In J. Feigenbaum and M. Merritt, editors, Distributed Computing and Cryptography, volume 2 of DIMACS Series in Discrete Mathematics and Theoretical Computer Science, pages 191-202. American Mathematical Society, 1991.

x_{11}	x_{12}	…	x_{1n}
x_{21}	x_{22}	…	x_{2n}
…	…	…	…
x_{n1}	x_{n2}	…	x_{nn}

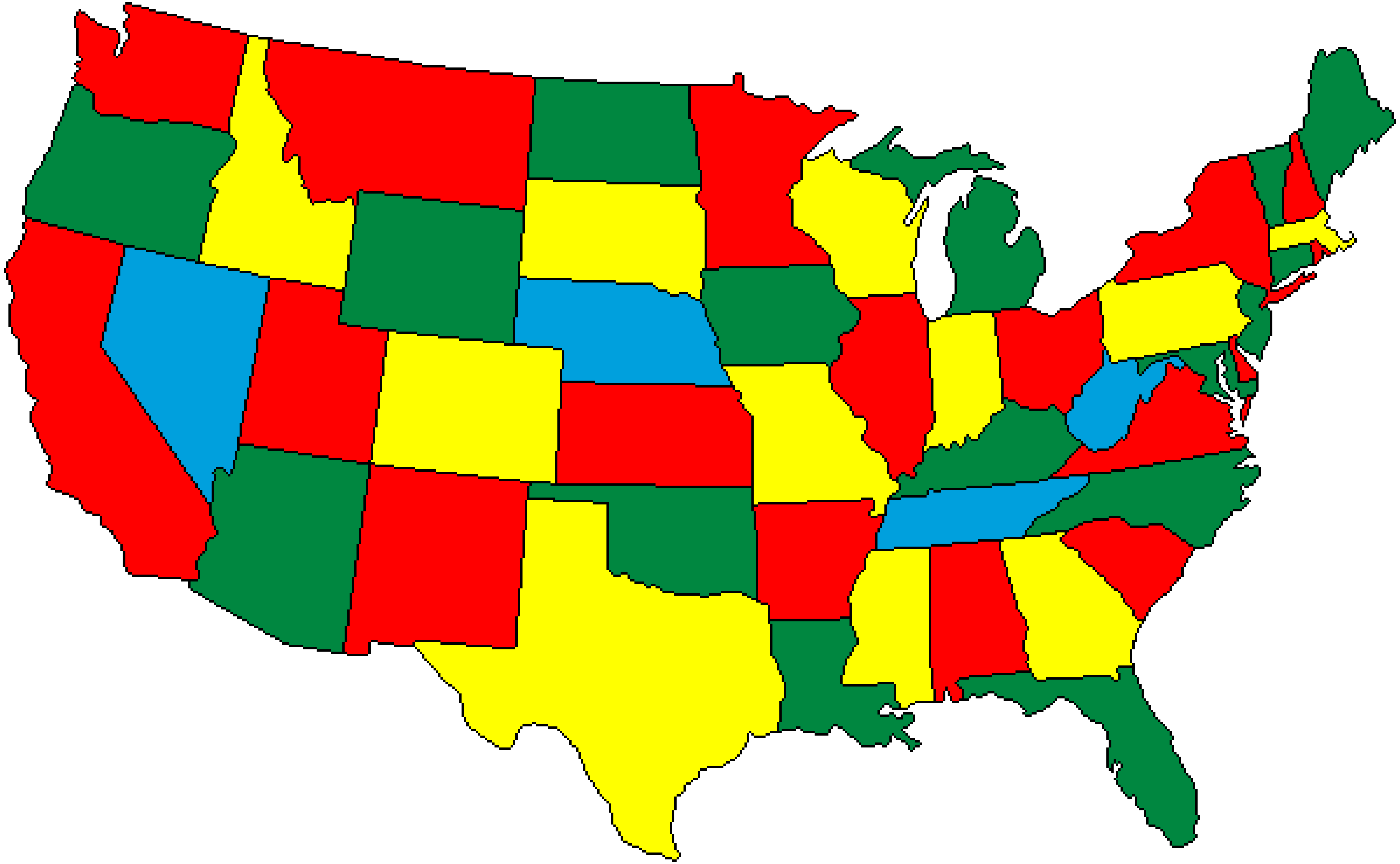
$$Det(X) = \sum_{\sigma} (-1)^{\sigma}\, x_{1\sigma(1)} x_{2\sigma(2)} \cdots x_{n\sigma(n)}$$

$$Perm(X) = \sum_{\sigma} x_{1\sigma(1)} x_{2\sigma(2)} \cdots x_{n\sigma(n)}$$

Algebraic Methods for Interactive Proof Systems

Carsten Lund*
Lance Fortnow†
Howard Karloff‡
University of Chicago

Noam Nisan§
Hebrew University



IP=PSPACE

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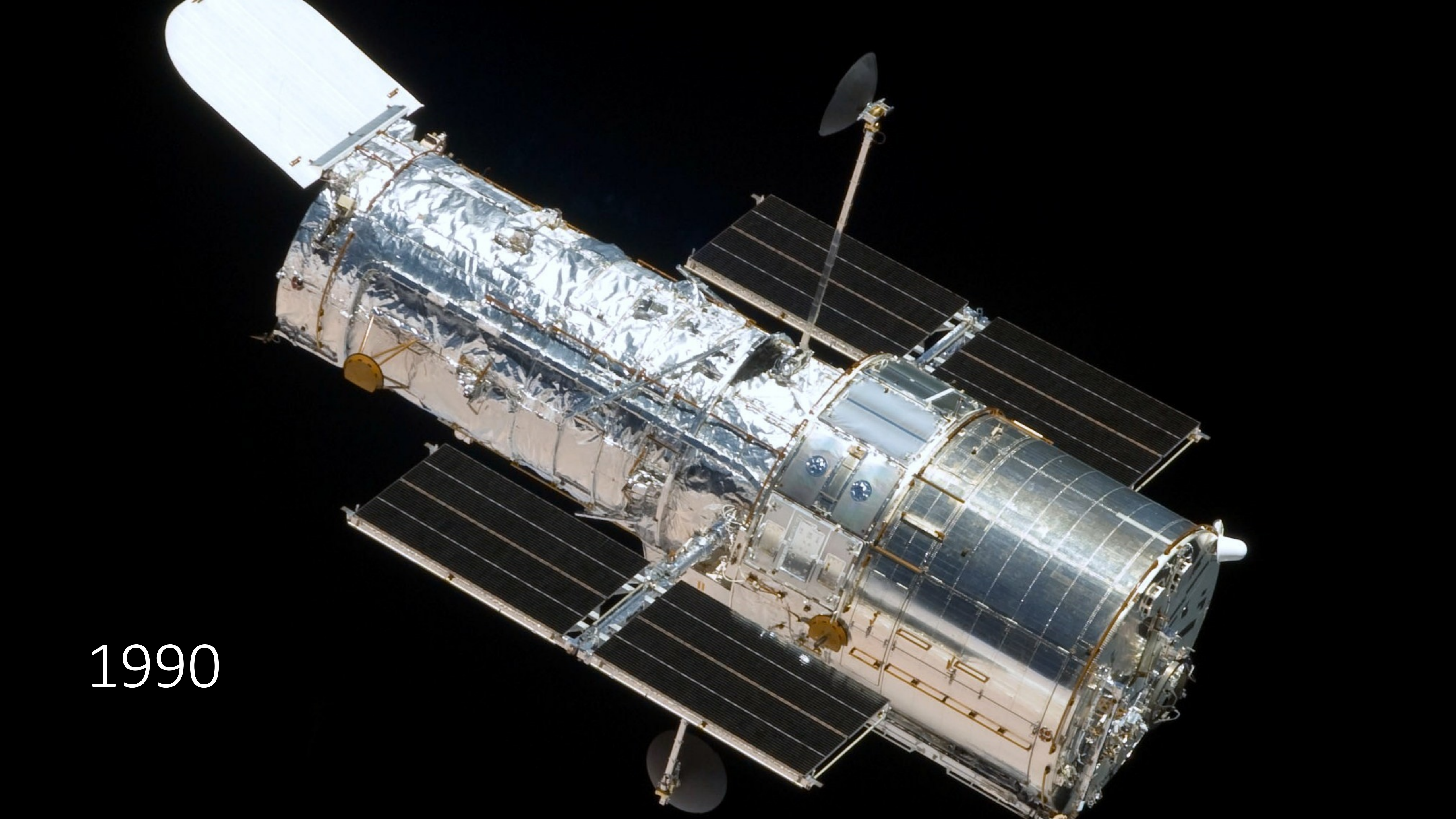
Rehovot, Israel


1990

Non-Deterministic Exponential Time has Two-Prover Interactive Protocols

László Babai[*†] Lance Fortnow[‡] Carsten Lund[§]

Interactive Proofs and the Hardness of Approximating Cliques

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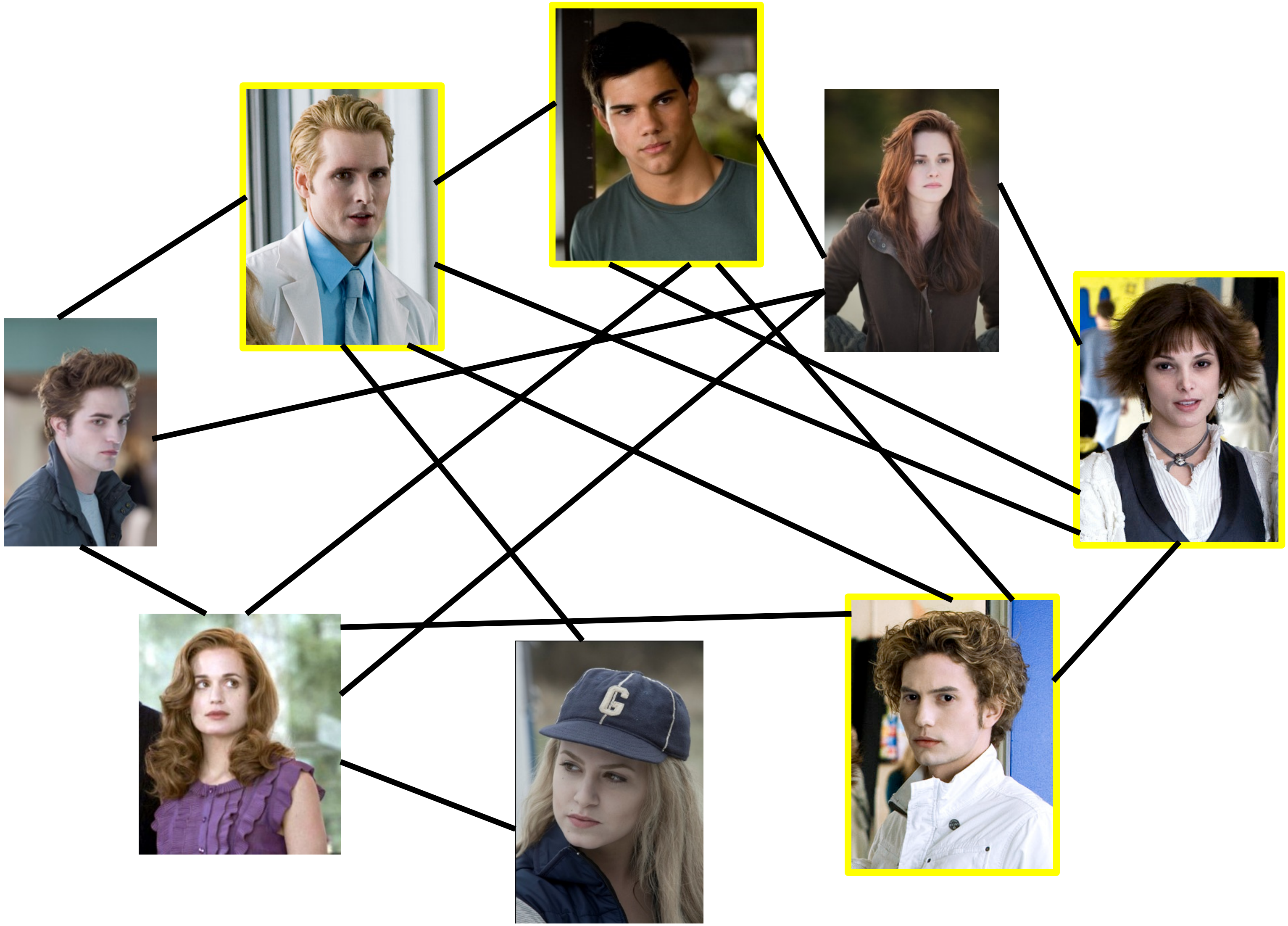

Proof Verification and the Hardness of Approximation Problems

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And More…

- Program Checking
  - Babai-Fortnow-Levin-Szegedy 1991
- Unique Games
  - Subhash Khot 2002
- Quantum Proof Systems
  - Anand Natarajan and John Wright 2019

SCIENTIFIC AMERICAN OCTOBER 1993

The Death of Proof

Computers are transforming the way mathematicians discover, prove and communicate ideas, but is there a place for absolute certainty in this brave new world?

By John Horgan