Compressed Sensing and Generative Models

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UT Austin

Talk Outline

1. Compressed sensing
2. Using generative models for compressed sensing
3. Learning generative models from noisy data

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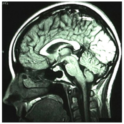
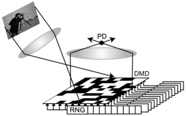


Medical Imaging

Astronomy



Single-Pixel Camera

Oil Exploration

Genetic Testing

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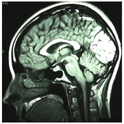

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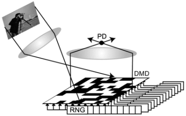

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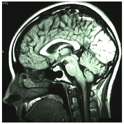

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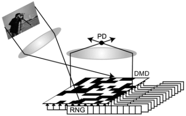

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- Measurements "incoherent" $\implies$ most info new.

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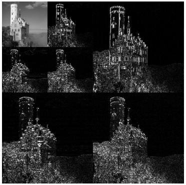
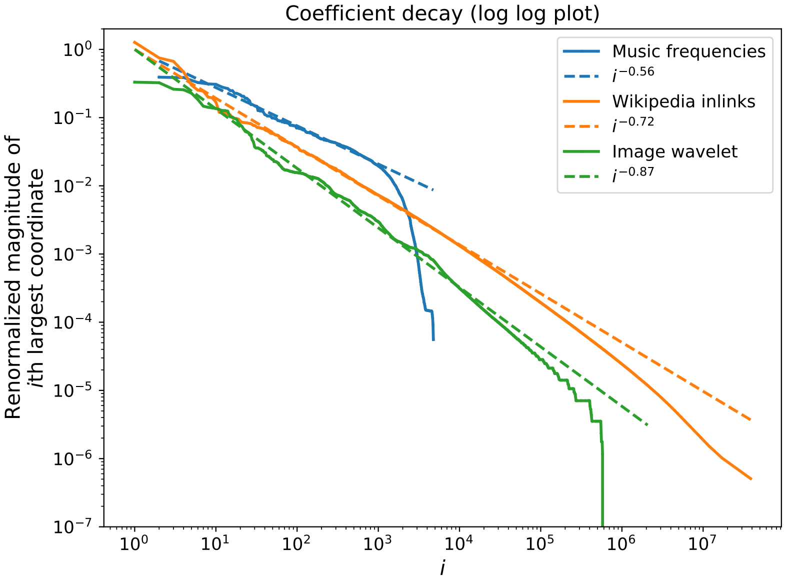
- Standard compressed sensing: *sparsity* in some basis



Approximate sparsity is common



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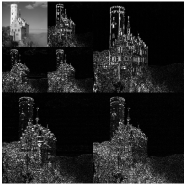

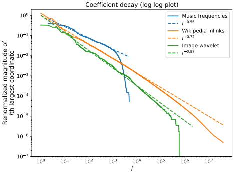

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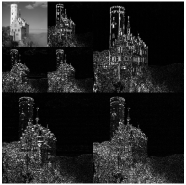

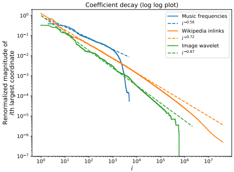

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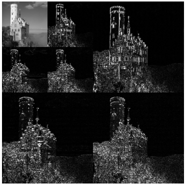

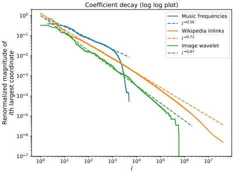

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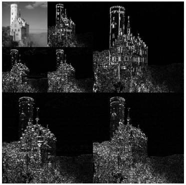

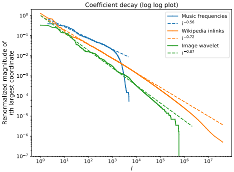

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- New info. per measurement is hopefully $\approx \log 100 = \Theta(1)$

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with high probability.

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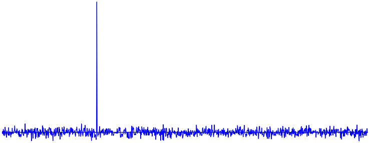
  - Such an $\widehat{x}$ can be found efficiently with, e.g., the LASSO.

Lower bound: $k = 1$

- Hard case: x is random e_i plus Gaussian noise w with $\|w\|_2 \approx 1$.



- Robust recovery must locate i.
- Observations $\langle v, x \rangle = v_i + \langle v, w \rangle = v_i + \frac{\|v\|_2}{\sqrt{n}} z$, for $z \sim N(0, 1)$.

1-sparse lower bound

P-Woodruff '11

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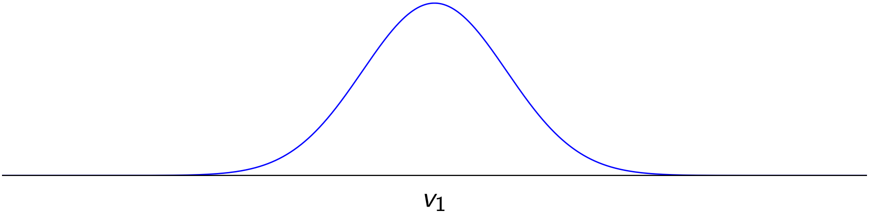

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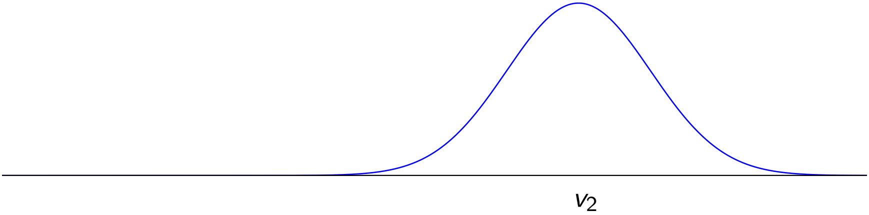

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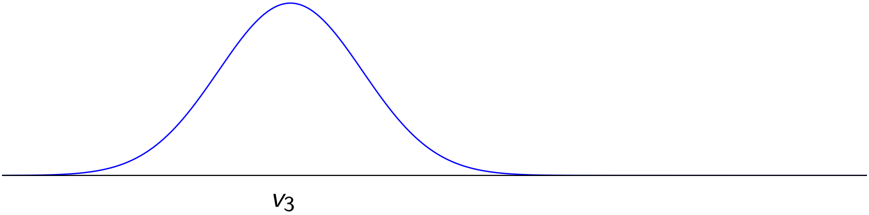

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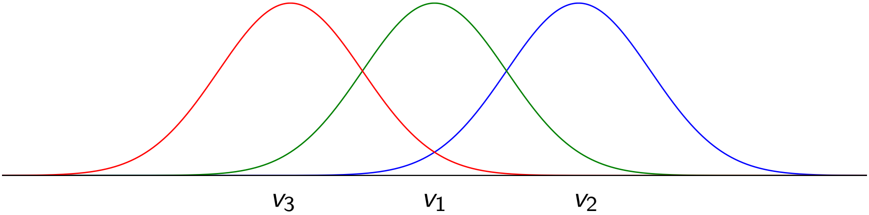

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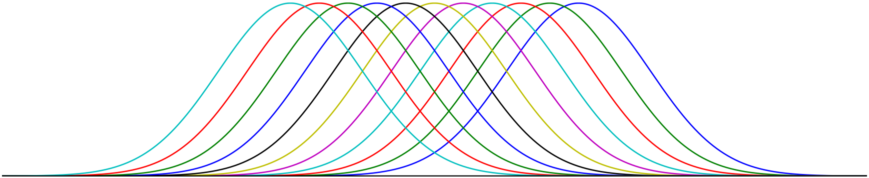

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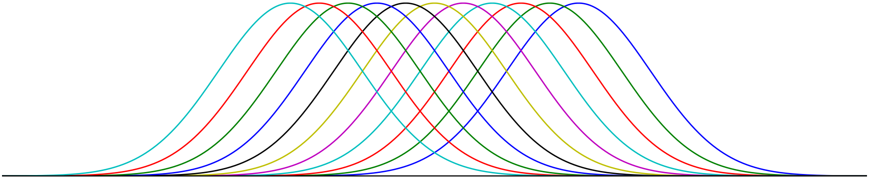

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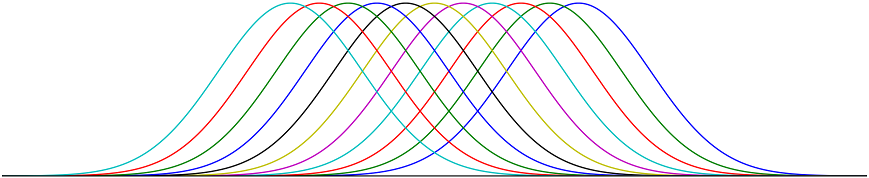

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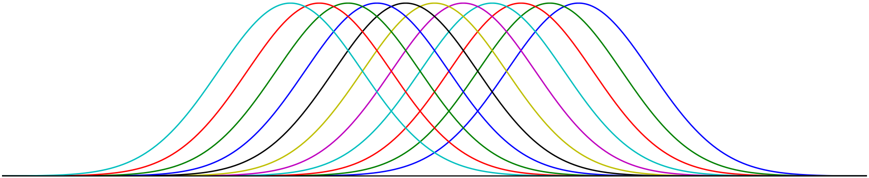

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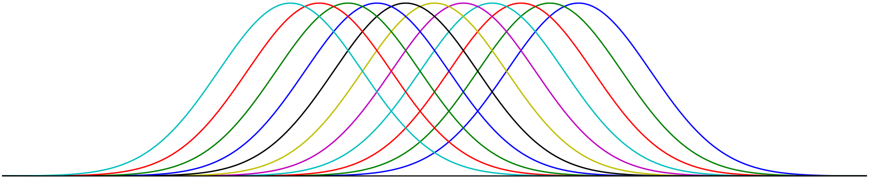

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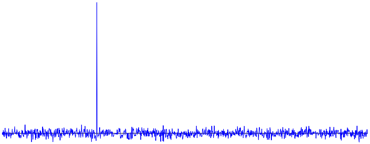

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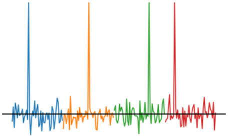

- General k: $m = \Omega(\log \binom{n}{k}) = \Omega(k \log(n/k))$.

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  - In particular: *generative models*.

Generative Models

Random
noise z

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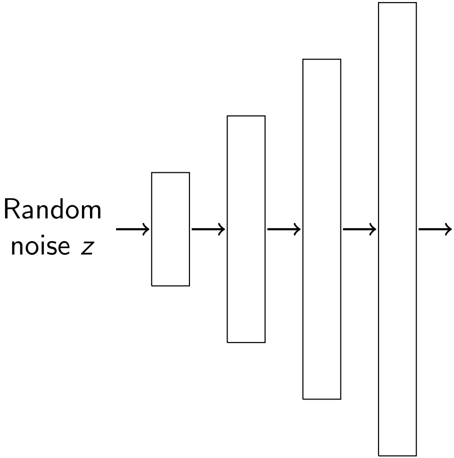

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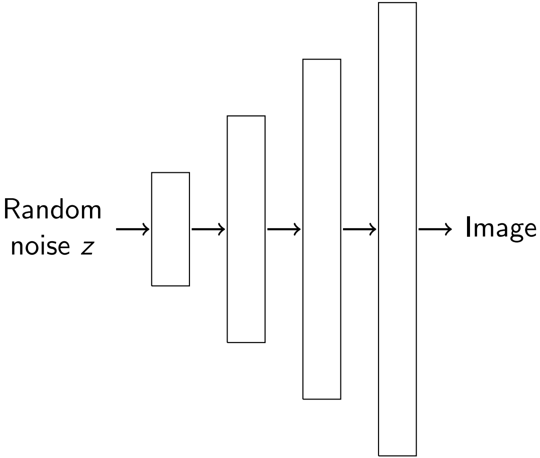

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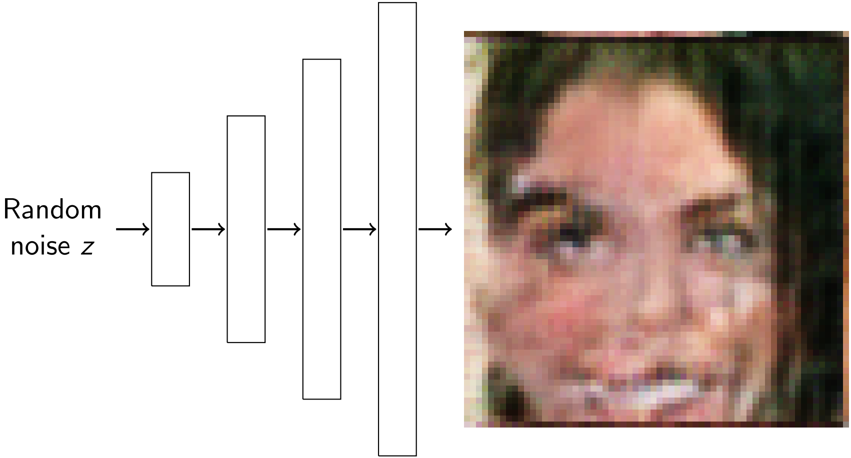

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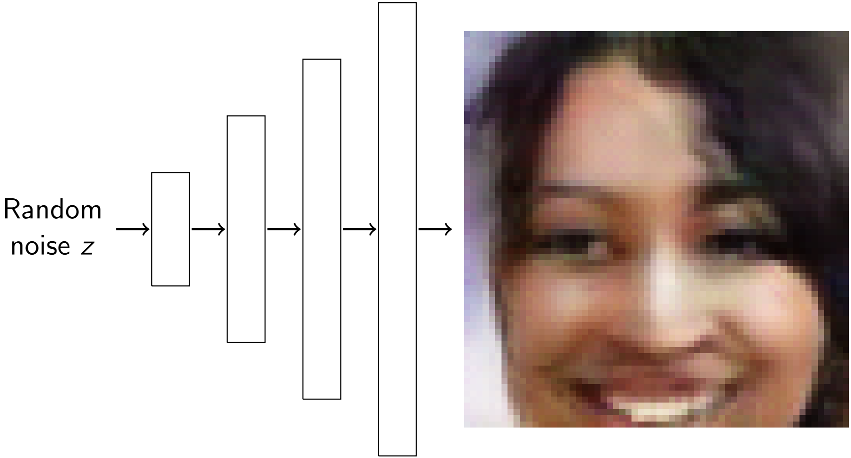

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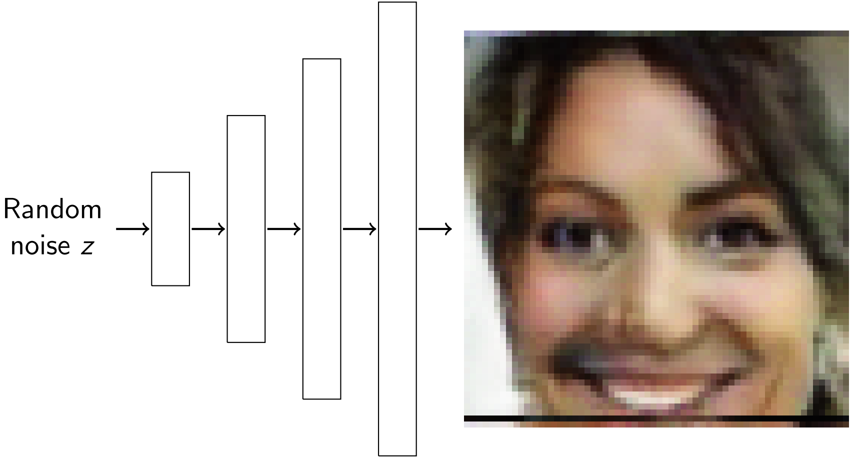

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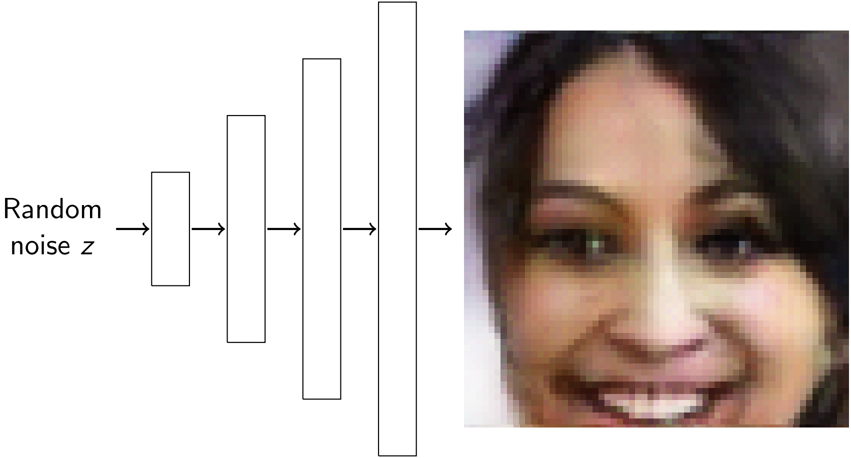

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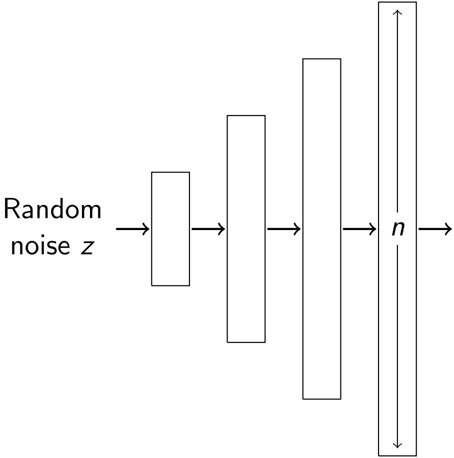

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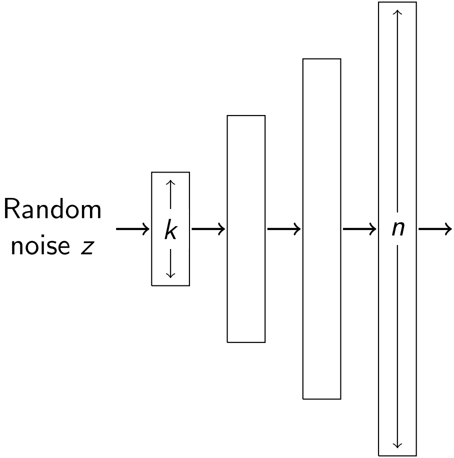

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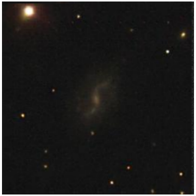

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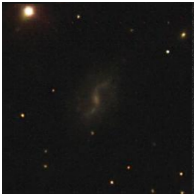

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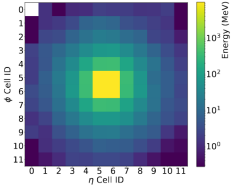

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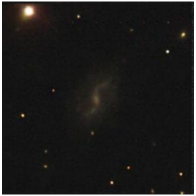

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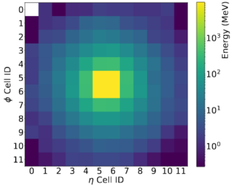

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Suggestion for compressed sensing

Replace "x is k-sparse" by "x is in range of $G : \mathbb{R}^k \to \mathbb{R}^n$".

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Our Results

"Compressible" = "near range(G)"

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  - In practice, optimization error seems negligible.

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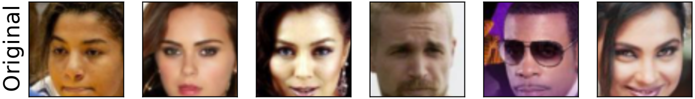
Experimental Results

Faces: $n = 64 \times 64 \times 3 = 12288$, $m = 500$



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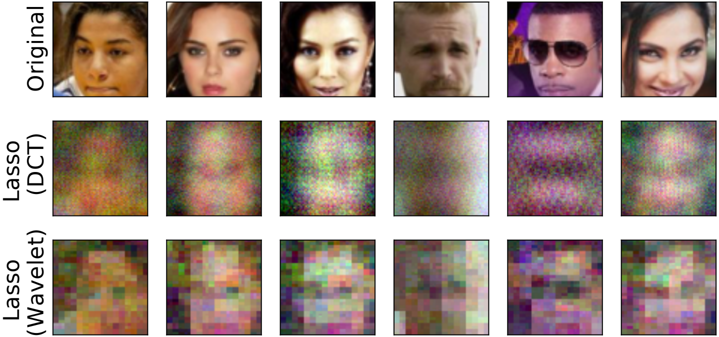

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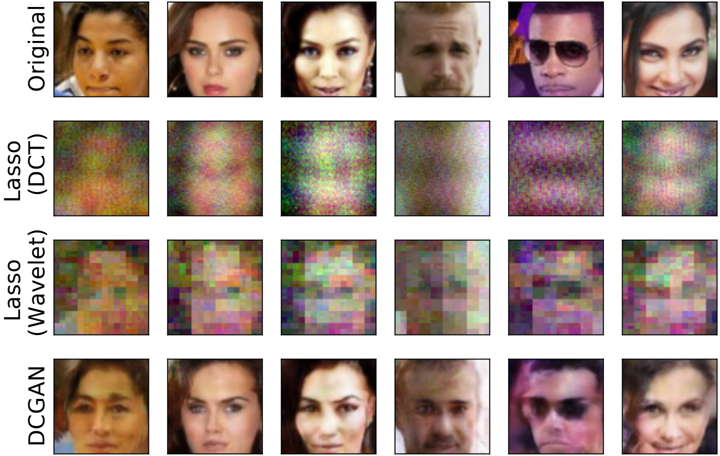

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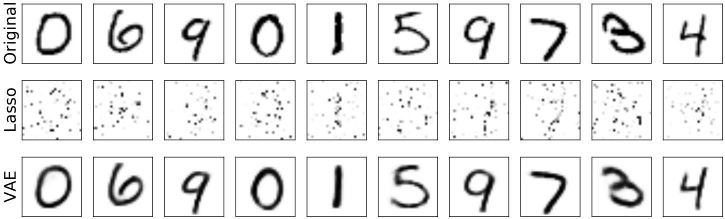
MNIST: $n = 28x28 = 784$, $m = 100$.



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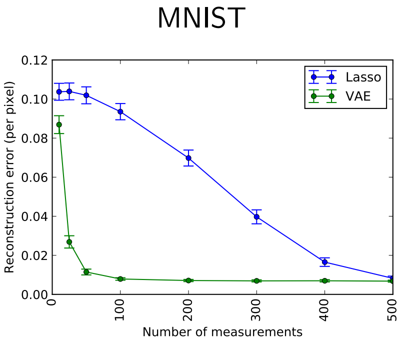

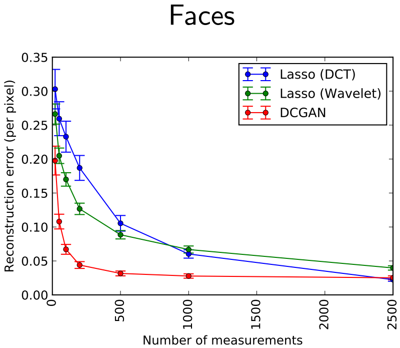

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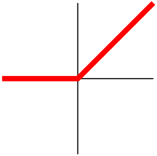

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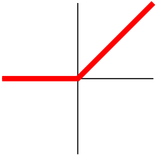

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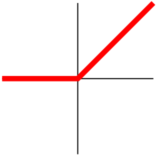

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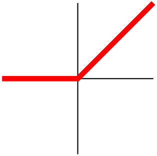

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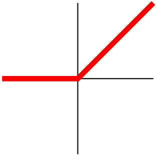

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- Induction: final output lies within n^{dk} k-dimensional hyperplanes.

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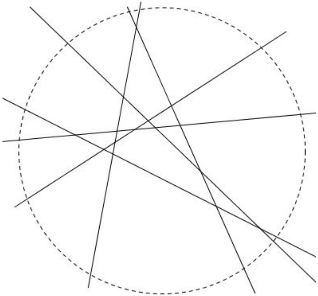

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Layer 1's output lies within a union of n^k k-dimensional hyperplanes.

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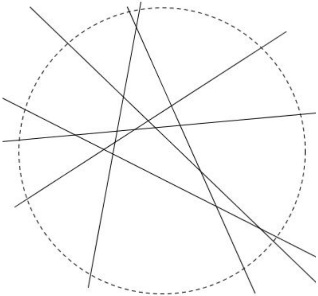

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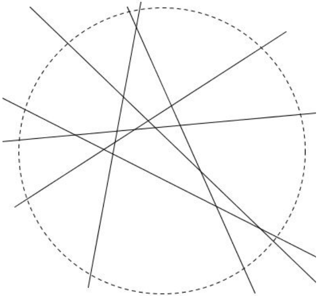

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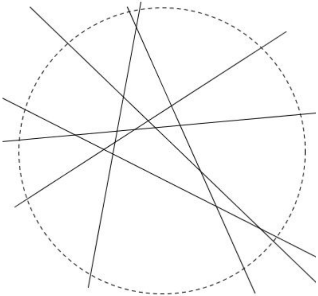

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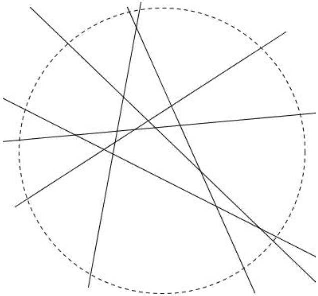

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Follow-up work

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For any L-Lipschitz $G : \mathbb{R}^k \to \mathbb{R}^n$, *recoving* $\widehat{x}$ *from* Ax *satisfying*

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- Provably fast for *random* networks (Hand-Voroninski '18)

Extensions

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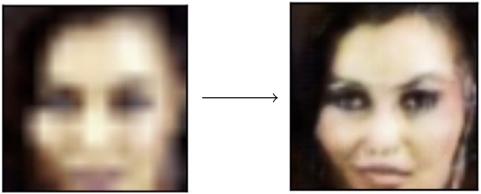

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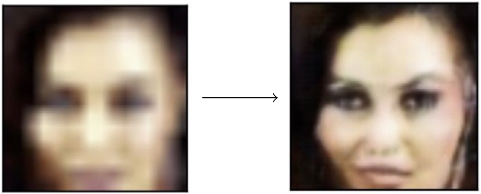

- Can apply even to nonlinear—but differentiable—measurements.

Talk Outline

1. Compressed sensing
2. Using generative models for compressed sensing
3. Learning generative models from noisy data

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Training from lots of data.

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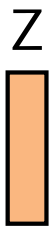
Question

Can we learn a GAN from incomplete, noisy measurements?

GAN Architecture



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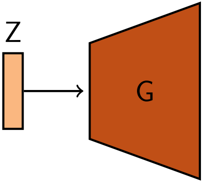

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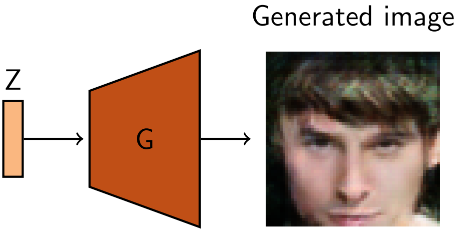

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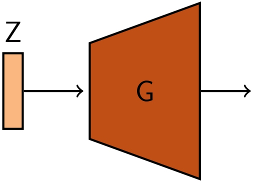
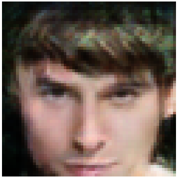


Generated image



Real image

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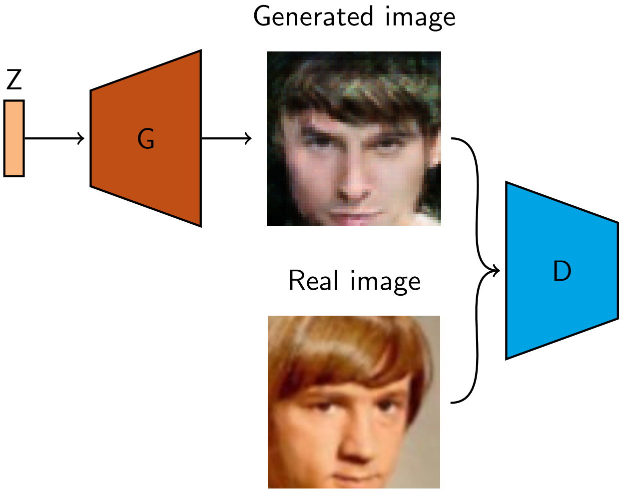

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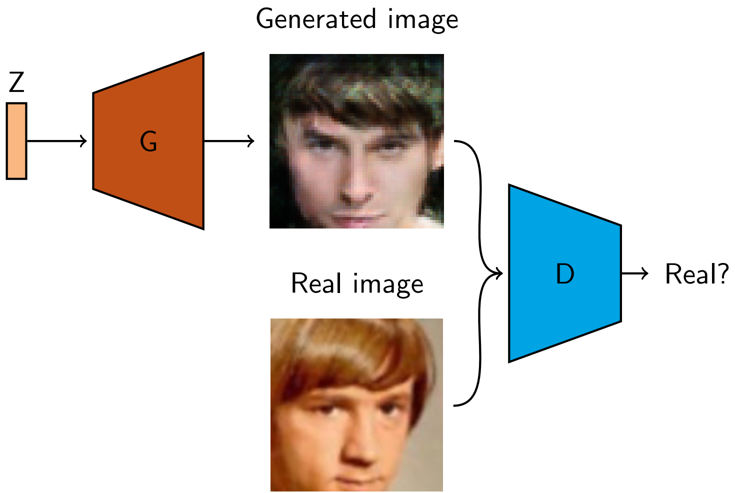

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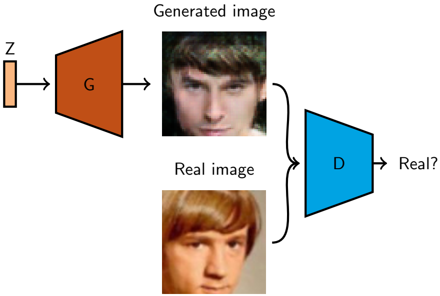

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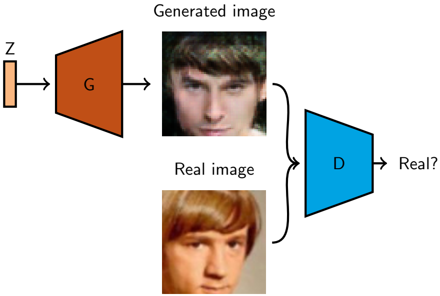

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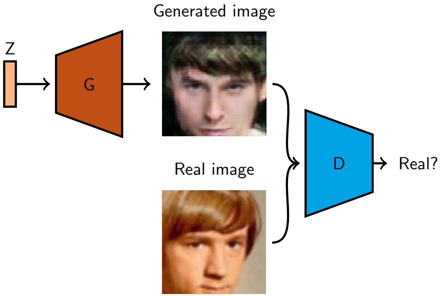

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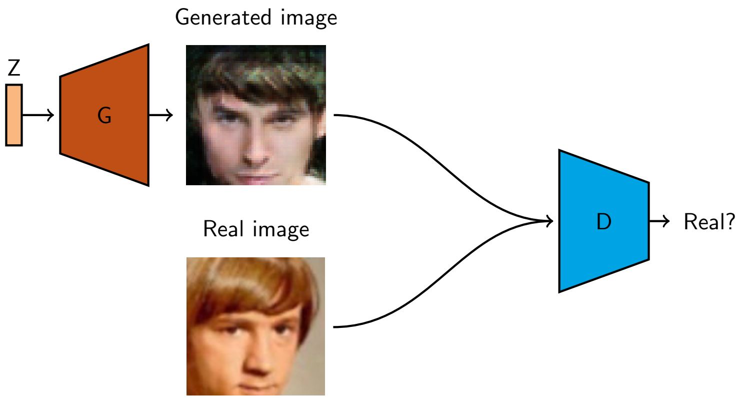
GAN training



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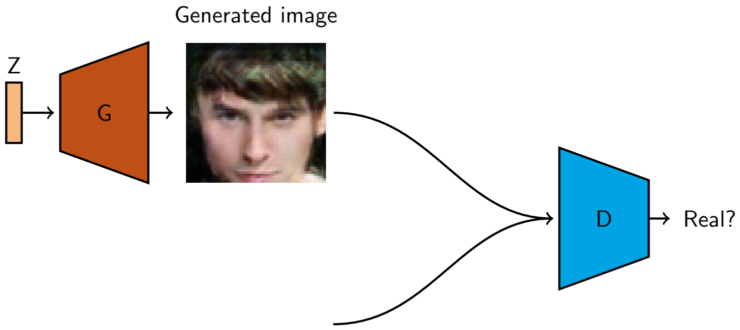

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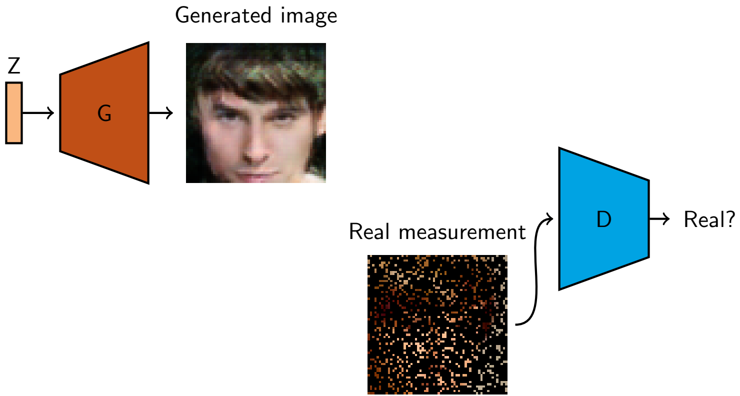

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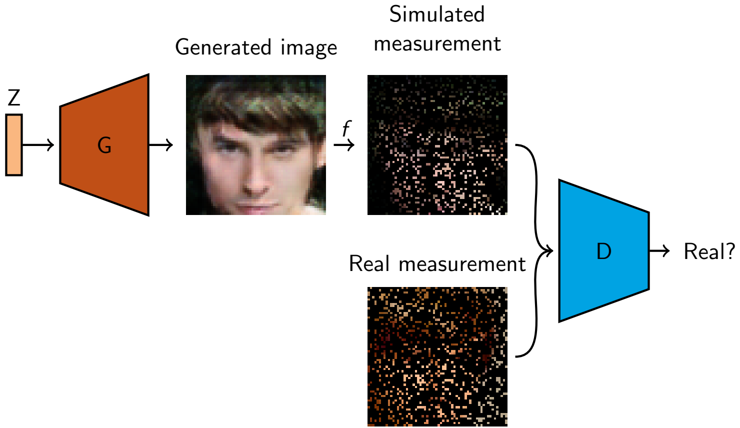

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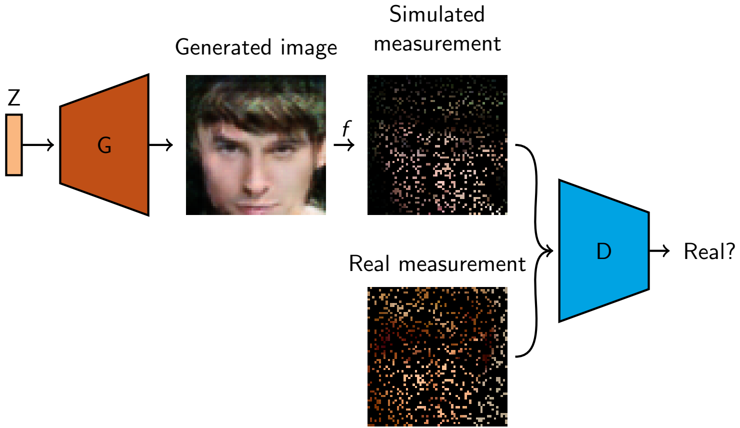

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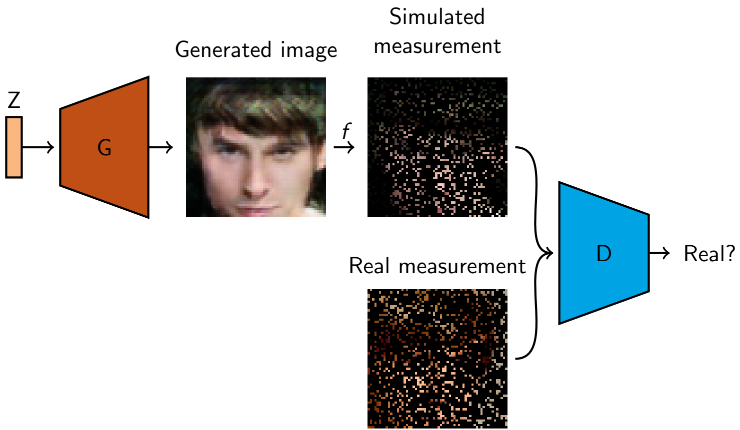

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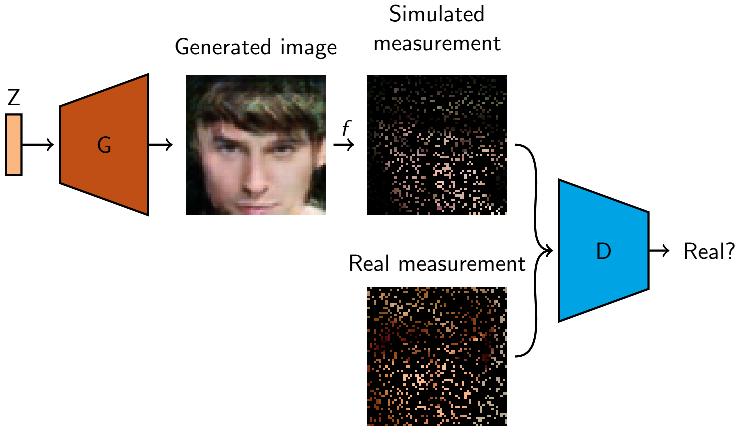

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- Can try this for any measurement process f you understand.
- Compatible with any GAN generator architecture.

Measurement: Gaussian blur + Gaussian noise

Measured

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Wiener Baseline

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		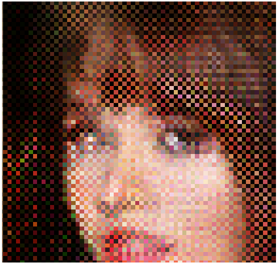

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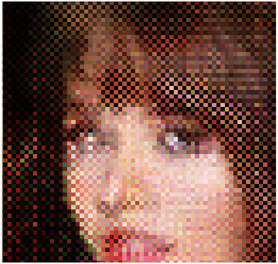

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Measurement: Obscured Square

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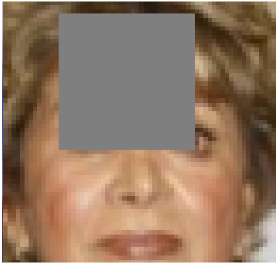

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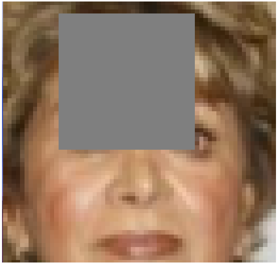

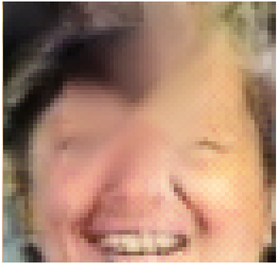

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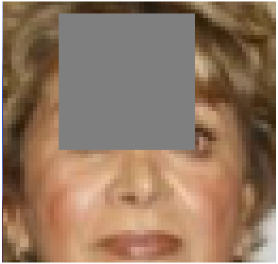

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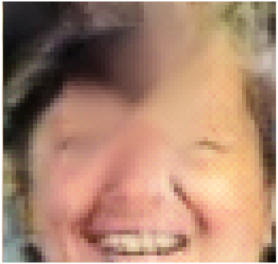

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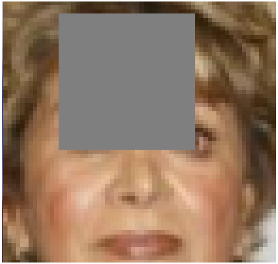

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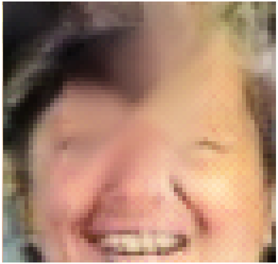

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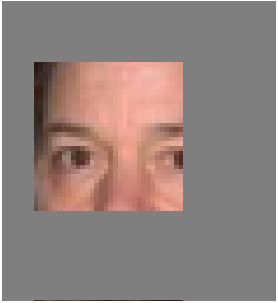

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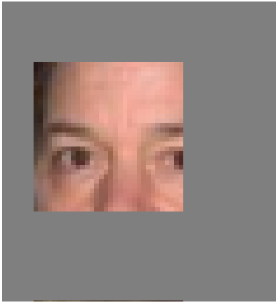

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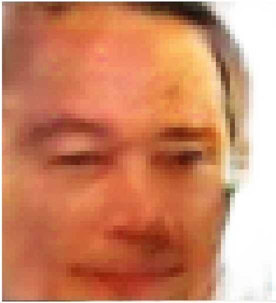
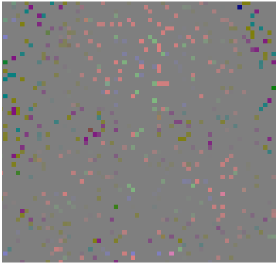


- Reveal a random square containing 25% of the image.
- AmbientGAN still recovers faces.

Measurement: Dropout

Measured



- Drop each pixel independently with probability $p = 95\%$.

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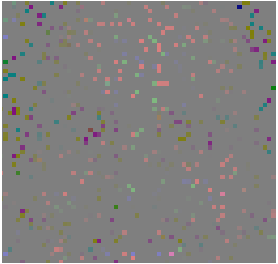
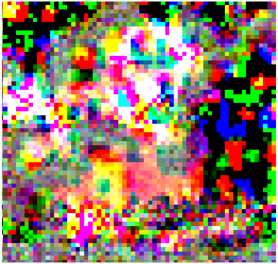
Measured



Blurring Baseline



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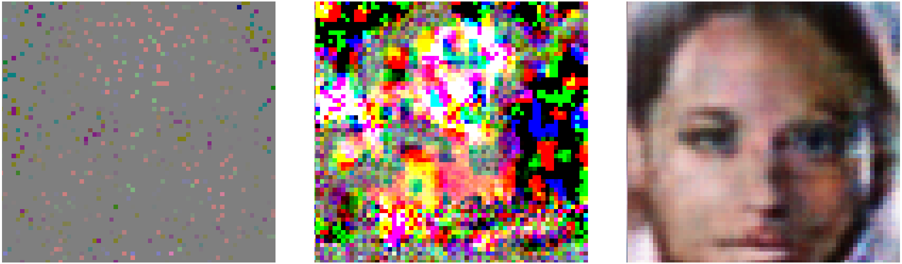
Measurement: Dropout

Measured | Blurring Baseline | AmbientGAN



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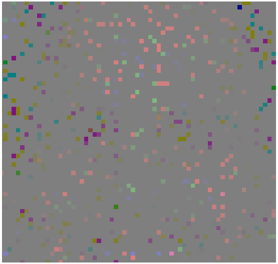

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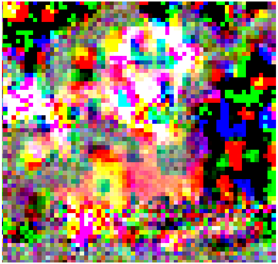

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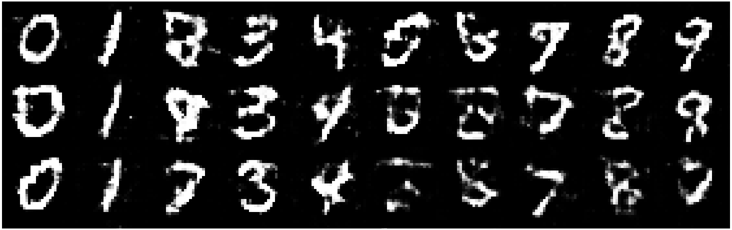

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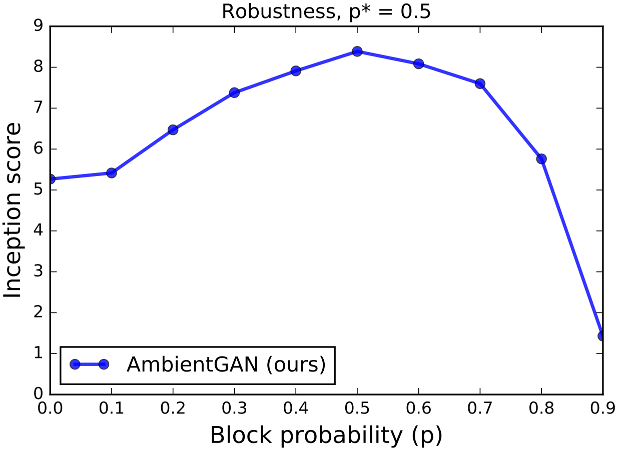

Compressed sensing

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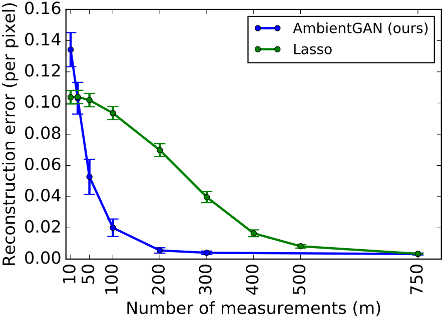

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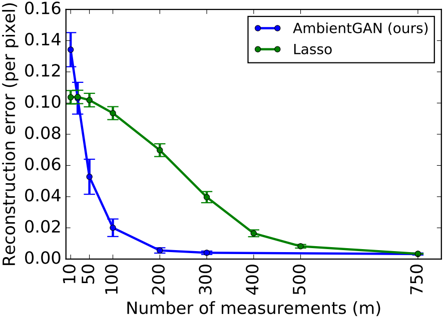
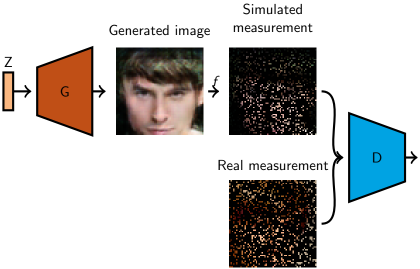
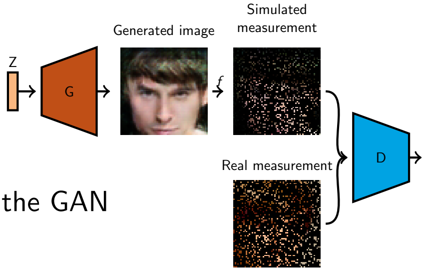


- Theorem about unique Nash equilibrium in the limit.

AmbientGAN


Simulated measurement
Generated image
Z
G
f
D
Real measurement

AmbientGAN



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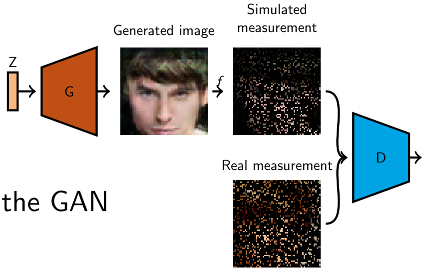

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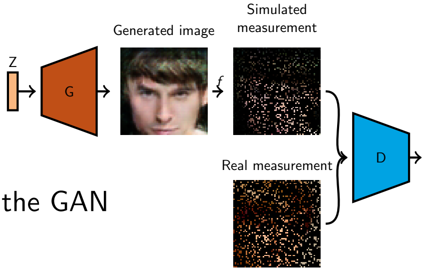

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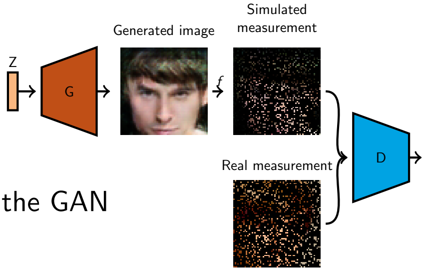

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- Could let us learn distributions we have no data for.
- Read the paper ("AmbientGAN") for lots more experiments.

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Thank You